Lecture Notes

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Course Title: GENERAL MATHEMATICS

Course Goal:

Class & Semester: HTC YEAR 1, SEMESTER 2 **Course Status: Compulsory**

Course Code: HTC1S **Credit Hours: 2**

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Topic:
SIMULTANEOUS EQUATION

Week : 1-2 **Lecture No.:1**

Learning Objective: By the end of the lectures students will be able to;
-Solve problems on simultaneous equation

Questions:

Solve the following equations for x and y, (a) by substitution, and (b) by elimination:

$x + 2y = -1$……. (1)
$4x - 3y = 18$ …..(2)

Notes: Simultaneous equations

Only one equation is necessary when finding the value of a **single unknown quantity.**

However, when an equation contains **two unknown quantities** it has an infinite number of solutions. When two equations are available connecting the same two unknown values then a unique solution is possible. Similarly, for three unknown quantities it is necessary to have three equations in order to solve for a particular value of each of the unknown quantities, and so on.

Equations that have to be solved together to find the unique values of the unknown quantities, which are true for each of the equations, are called **simultaneous equations**.

Two methods of solving simultaneous equations analytically are used in this write up but there are more methods:

(a) By **substitution**, and (b) by **elimination**..

(b)

SOLUTION

(a) **By substitution**

From equation (1): $\boldsymbol{x=-1-2y}$

Substituting this expression for x into equation (2)

gives: $4(-1 - 2y) - 3y = 18$

This is now a simple equation in y.

Removing the bracket gives:

$$-4 - 8y - 3y = 18$$
$$-11y = 18 + 4 = 22$$
$$y = \frac{22}{-11}$$
$$y = -2$$

Substituting $y=-2$ into equation (1) gives:

	$x + 2(-2) = -1$ $x - 4 = -1$ $x = -1 + 4 = 3$ **Thus x=3 and y=−2 is the solution to the simultaneous equations.** (Check: In equation (2), since x =3 and y=−2, LHS=4(3)−3(−2)=12+6=18=RHS) (b) **By elimination** $x + 2y = -1$.................. (1) $4x - 3y = 18$................ (2) If equation (1) is multiplied throughout by 4 the coefficient of x will be the same as in equation (2), giving: $4x + 8y = -4$.............. (3) Subtracting equation (3) from equation (2) gives: $4x - 3y = 18$ (2) $4x + 8y = -4$ (3) ——————— $0 - 11y = 22$ ——————— Hence $y = \frac{22}{-11}$ $y = -2$ (Note, in the above subtraction, 18−(−4) = 18+4=22 Substituting y=−2 into either equation (1) or equation(2) will give x =3 as in method (a). The solution ***x*=3, *y*=−2** is the only pair of values that satisfies both of the original equations.
Problem 2. Solve, by a substitution method, the simultaneous equations: $3x - 2y =$ 12......................(1) $x + 3y =$ −7....................... (2)	**SOLUTION** From equation (2), $x=-7-3y$ Substituting for x in equation (1) gives: $3(-7 - 3y) - 2y = 12$ i.e. $-21 - 9y - 2y = 12$ $-11y = 12 + 21 = 33$ Hence $y=\frac{33}{-11}$ $y=-3$ Substituting y=−3 in equation (2) gives: $x + 3(-3) = -7$ i.e. $x - 9 = -7$ Hence $x = -7 + 9 = 2$ Thus **x=2, y=−3** is the solution of the simultaneous equations. (Such solutions should always be checked by substituting values into

each of the original two equations.)

Problem 3. Use an elimination method to solve the simultaneous equations:

$3x + 4y = 5$

$2x - 5y = -12$

SOLUTION

If equation (1) is multiplied throughout by 2 and equation (2) by 3, then the coefficient of x will be the same in the newly formed equations.

Thus 2 × equation (1) gives:

$6x + 8y = 10$...............(3)

3 × equation (2) gives:

$6x - 15y = -36$........... (4)

Equation (3) - equation (4) gives:

$0 + 23y = 46$

i.e. $y = \frac{46}{23}$

$y = 2$

(Note $+8y - -15y = 8y + 15y = 23y$ and $10 - (-36) = 10 + 36 = 46$.

Alternatively, 'change the signs of the bottom line and add'.)

Substituting $y = 2$ in equation (1) gives:

$3x + 4(2) = 5$

from which $3x = 5 - 8 = -3$

$3x = -3$

and $x = -1$

Checking in equation (2),

left-hand side

$2(-1) - 5(2) = -2 - 10 = -12$

right-hand side.

Hence $\boldsymbol{x = -1}$ **and** $\boldsymbol{y = 2}$ is the solution of the simultaneous equations.

The elimination method is the most common method of solving simultaneous equations.

Problem 4. Solve :

$7x - 2y = 26$......... (1)

$6x + 5y = 29$......... (2)

When equation (1) is multiplied by 5 and equation (2) by 2 the coefficients of y in each equation are numerically the same, i.e. 10, but are of opposite sign.

5 × equation (1) gives:

$35x - 10y = 130$............. (3)

2 × equation (2) gives:

$12x + 10y = 58$............... (4)

$$\begin{array}{l} 35x - 10y = 130 \\ 12x + 10y = 58 \\ \hline 47x + 0 = 188 \end{array}$$ ....(5)

Hence $x = \frac{188}{47}$

$x=4$

[Note that when the signs of common coefficients are different the two equations are added, and when the signs of common coefficients are the same the two equations are subtracted (as in Problems 1 and 3).]

Substituting $x=4$ in equation (1) gives:

$7(4) - 2y = 26$

$28 - 2y = 26$

$28 - 26 = 2y$

$2 = 2y$

Hence $y = 1$

Checking, by substituting $x=4$ and $y=1$ in equation (2), gives:LHS

$6(4) + 5(1) = 24 + 5 = 29 = \text{RHS}$

the simultaneous equations.
The elimination method is the most common method of solving simultaneous equations.

Problem 5. Solve :

$7x - 2y = 26$........ (1)

$6x + 5y = 29$........ (2)

SOLUTION

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Hence $x = \frac{188}{47}$

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[Note that when the signs of common coefficients are different the two equations are added, and when the signs of common coefficients are the same the two equations are subtracted (as in Problems 1 and 3).]

Substituting $x=4$ in equation (1) gives:

$7(4) - 2y = 26$

$28 - 2y = 26$

$28 - 26 = 2y$

$2 = 2y$

Hence $y = 1$

	Checking, by substituting $x = 4$ and $y=1$ in equation (2), gives:LHS $6(4) + 5(1) = 24 + 5 = 29$ = RHS equation (2) gives: LHS = $4(-2) + (-3) + 11 = -8 - 3 + 11= 0$ = RHS **Hence the solution is $p=-2$, $q=-3$**
Problem 6. Solve $\frac{x}{8} + \frac{5}{2} = y$ ....(1) $13 - \frac{y}{3} = 3x$ ....(2)	**SOLUTION** Whenever fractions are involved in simultaneous equation it is usual to firstly remove them. Thus, multiplying equation (1) by 8 gives: $\frac{8x}{8} + \frac{40}{2} = 8y$ ....(3) $39 - \frac{3y}{3} = 9x$ ....(4) rearrange the equations and simplify $x-8y=-20$ ....(5) $9x+y= 39$..................(6) Multiplying equation (6) by 8 gives $x-8y=-20$ $\underline{72x+8y=312}$ $73x+0 = 292$ $x=\frac{292}{73}$ $x=4$ substituting the value of x=4 in equation 5 $4-8y=-20$ $-8y = -24$ $y= \frac{-24}{-8}$ $y=3$ Hence the solution of equations are x=4 and y=3
Problem 7. Solve $2.5x + 0.75 - 3y = 0$ $1.6x = 1.08 - 1.2y$	**SOLUTION** It is often easier to remove decimal fractions. Thus multiplying equations (1) and (2) by 100 gives: $250x + 75 - 300y = 0$.......... (1) $160x = 108 - 120y$................ (2) Rearranging gives: $250x - 300y = -75$............ (3) $160x + 120y = 108$ ....(4) Multiplying equation (3) by 2 gives: $500x - 600y = -150$ (5) Multiplying equation (4) by 5 gives: $800x + 600y = 540$ (6) Adding equations (5) and (6) gives: $500x - 600y = -150$ $\underline{800x + 600y = 540}$ $1300x + 0 = 390$

	x = 390/1300 **x= 39/130** **x= 3/10** **x= 0.3** Substituting x =0.3 into equation (1) gives: 250(0.3) + 75 − 300y = 0 75 + 75 = 300y 150 = 300y **y = 150/300** **y= 0.5** Checking x =0.3, y=0.5 in equation (2) gives: LHS = 160(0.3) = 48 RHS = 108 − 120(0.5) = 108 − 60 = 48 Hence the solution is x=0.3, y=0.5

Summary: The concept discuss in this are intended to encourage people teaching and learning of mathematics to be able to Calculate problems on simultaneous linear equation

Assignment:

Solve the following simultaneous equations and verify the results.

1. $7p+11+2q=0$
   -1=3q-5p [**p=-1, q=-2**]
2. $\frac{x}{2} + \frac{y}{3} = 4$
   $\frac{x}{6} - \frac{y}{9} = 0$ **[x=4, y= 6]**
3. 3a – 4b = 5
   5a – 2b = -15 [**a=-5, b=-5]**

Mammy Iye and Nenneh are in m and n years old respectively. The sum of their ages is 44. Mammy Iye is eight years older than Nenneh. How old is each of them.

Further Reading: Solving quadratic equations

References:

College Algebra, 8th Edition, Ron Larson, Brooks Cole, ISBN:14394869X. College Algebra and trigonometry, 7th Edition, Richard N Aufmann and Vernon C Barker, Brooks Cole, ISBN:1439048606